

Numerical Study of Hydromagnetic Natural Convection in Moving Vertical Surface Passes through Porous Medium

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Abstract—The numerical investigation is done in the present work for the double diffusive natural convective flow in vertical moving surface in Darcy porous medium. The finite difference method is adopted for the numerical study. The effect of prandtl number, magnetic number and schmidt number is depicted through graphical representation. The skin friction coefficient is also calculated to understand the resistant force exerted on an object moving in a fluid.

1. INTRODUCTION

The Fluid flow with convective heat and mass transfer through a porous medium has been studied in the past decades as it has applications to thermal insulation, nuclear waste disposal, solid matrix heat exchanger and other practical applications. Natural convective flows are always in physical and engineering problems such as chemical catalytic reactors, nuclear waste material etc. There are many engineering applications in the heat and mass transfer simultaneously from different geometry embedded in porous medium such as porous concrete, thermal insulation, increased oil recovery, cooling of nuclear reactors, and underground energy transport. In recent years, the potential use of the MHD is to influence the flow current of the electrically operated fluid for the purpose of thermal protection, braking, propulsion and control. From the point of the application, the model study on the effect of the magnetic field on convection flow has been done by many investigators. Technically, the problems of MHD convection in the field of aeronautics, chemical engineering and electronics are also very important. Model studies of the above events of MHD convection have been done by many researchers. Convection in porous media contains applications in flow through oil extraction, thermal energy storage and filtering equipment.

The fundamental importance of convective flow in porous media has been well-documented in the recent book by Ingham and Pop[1], Nield and Bejan[2].

It is observed from the literature that many researchers have already worked on free convection flow past a semi-infinite vertical plate. Some of the details are given below.

Kapoor et al. [4] have attempted the analytical solution of MHD free convective flow through vertical flat plate in porous medium in addition of that Rawat. et.al [5] shown that effect of MHD flow, heat and mass transfer of a micropolar fluid over a nonlinear stretching sheet with variable micro inertia density, heat flux and chemical reaction in a non-Darcy porous medium were observed. Goud et al.[6] have represented a mathematical pattern for the radiation and Soret effects on mass transfer flow through a highly porous medium with chemical reaction and heat generation. Umamaheshwar [7] numerical investigated MHD free convection flow of a non-Newtonian fluid past an impulsively started vertical plate in the presence of thermal diffusion and radiation absorption. However, Uddin et.al.[8] studied the effect of thermal radiation along with heat generation/absorption on MHD free convective flow of a micropolar fluid. Ibrahim et al. [9] studied effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Rawat and Kapoor[10] initiate the numerical solution of linearly moving permeable vertical surface in presence of magnetic field by using FEM, their work is restricted to viscous media. Keeping in view the above and work of [10] an attempt is being taken here to do the numerical investigation of linearly moving permeable vertical surface in presence of magnetic field in porous medium using finite difference method.

2. MATHEMATICAL MODEL

It is assumed that the Double- diffusive free convection in steady laminar boundary layer flow of an incompressible viscous fluid along with linearly moving permeable vertical surface under the action of the transverse magnetic field takes place in porous medium. The velocity of the fluid far away

from the plate is equal to zero i.e the flow is basically MHD flow. The variation of surface temperature and concentration are linear. The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is neglected. No electric field is assumed to exist and magnetic dissipations are assumed to be negligible. Besides this, the Hall Effect, viscous dissipation and the joule heating rms are also not considered. The fluid is considered to be gray: absorbing-emitting radiation but non-scattering medium. The chemical reaction takes place during mass transfer. The external heat source is taken into consideration. Under these assumptions along with boussinesq approximations, the boundary layer equations for the above problem can be formulated as.

3. GOVERNING EQUATIONS:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \mu \left(\frac{\partial^2 u}{\partial y^2} \right) + g[\beta_T(T - T_\infty) + \beta_S(S - S_\infty)] - \frac{\sigma_0 B_0^2}{\rho} u - \frac{\mu}{K} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \sigma \frac{\partial^2 S}{\partial y^2} - \Gamma(S - S_\infty) \tag{4}$$

Subject to the boundary conditions

$$y \rightarrow \infty: S = S_\infty, T = T_\infty, u = 0 \tag{5}$$

$$y = 0: v = -v_w, S = S_\infty + bx, T = T_\infty + ax, u = Bx \tag{6}$$

where u and v are velocity components in X and Y directions respectively, T denotes fluid temperature, g is acceleration due to gravity, μ is kinematic viscosity, C is species concentration, σ is electrical conductivity, B₀ is externally imposed magnetic field in the Y-direction. K is permeability, Γ is chemical reaction parameter.

Similarity Transformation

Using the normalized variables:

$$u = \frac{\partial \psi}{\partial Y}, v = -\frac{\partial \psi}{\partial X}, \eta = \sqrt{\frac{B}{\mu}} Y, f(\eta) = \frac{\psi}{X\sqrt{B\mu}}, \theta(\eta) = \frac{T - T_\infty}{T_w(X) - T_\infty}, c(\eta) = \frac{C - C_\infty}{C_w(X) - C_\infty} \tag{7}$$

Non-Dimensional Governing Equation

$$F' = U, \tag{8}$$

$$U'' + FU' - \left(M + \frac{1}{Da} + U \right) U + \frac{Gr_T}{Re^2} [\theta + N_c] = 0, \tag{9}$$

$$\theta'' + Pr(F\theta' - U\theta) = 0, \tag{10}$$

$$C'' + Sc(FC' - UC + \chi C) = 0 \tag{11}$$

with the boundary conditions:

$$\eta = 0, F = F_w, U = 1, \theta = 1, \varphi = 1; \tag{12}$$

$$\eta \rightarrow \infty, U = 0, \theta = 0, \varphi = 0 \tag{13}$$

Where, Magnetic parameter is given by $M = \frac{\sigma B_0^2}{\rho B}$,

Prandtl number (Pr) is given by $= \frac{\mu}{\alpha}$,

Chemical reaction parameter(χ) is given by $= \frac{\kappa c}{B}$,

Buoyancy parameter is given by $\frac{Gr_T}{Re^2} = \frac{g\beta_T a}{B^2}$,

Buoyancy ratio is given by $N_r = \frac{\beta_{cb}}{\beta_{Ta}}$,

Schmidt number is given by $= \frac{\mu}{D}$,

whereas D is coefficient of diffusion in the mixture.

4. NUMERICAL COMPUTATION

The dimensionless equations (8) – (11) together with the non dimensional boundary conditions (12)-(13) are solved numerically by means of the second order accurate finite difference technique with order of accuracy of 0.0005. The applied technique is well documented in the book of Numerical solution of differetial equation by Jain [3].

5. RESULT & DISCUSSION

To understand the flow dynamics (The dimensionless velocity, temperature and concentration distributions) of Double diffusive free convective flow from a linearly moving permeable vertical surface in porous medium. The effect of physical parameters like Prandtl number, Schmidt number and magnetic number is presented in this work..

Before discussing the effect of these parameters the numerical solution validation is done in some special case. Table(1) is shown in order to compare the published results of M.M Abdelkhalek (2009) by calculating the wall temperature gradient θ'(0) for the different value of Pr (M = 0.72, 1.0, 3.0, 10.0 & 100. There has been found the good agreement between the published results.

At $F_w = 0$			
Pr	Abdelkhalek MM(2009)	Present work FDM	Relative Error
0.72	0.8086	0.8088	0.00024734
1.0	1.0000	1.0015	0.00150000
3.0	1.9246	1.9237	-0.00046763
10.0	3.7216	3.7225	0.0002418
100.0	12.294	12.294	0

Table -I
Comparison of Non-Dimensional temperature gradient θ''(0) at Different Values of M at $F_w = 0, Q = 0, Kr = 0, Da = 10^7$

To study the effect of prandtl number on flow profile, figure 2 (a), 2(b) and 2(c) is plotted for the different value of Pr =0.5, 0.71, 1, 2 while fixing the other parameters at Gr = 2.0, Gc =

2.0, $M = 1$, $Da = 0.05$, $\chi = 0.5$, $Sc = 0.6$. This has been observed that as the Prandtl number increased with reference to kinematic viscosity the velocity is keep on decreasing which almost flat for the characteristic length $\eta \in (5,7)$. Furthermore, this has been observed that the temperature is also being decreases on higher values of Pr. This shows that the thermal boundary-layer becomes thinner for the higher values of Pr. However, there is a sudden change in concentration profile on higher values of Prandtl number. Which shows that the at very near to the wall the mass flux is much effected in respect to the far away from the boundary. The figure 2(c) also shows that the effect of Pr is significant on concentration profile.

In order to study the behavior of fluid flow profile in respect to the Magnetic number (M) figure 3(a) to 3(c) is plotted. The M is taken 0.5, 1, 1.5, 2, 2.5, 3 while fixing the other parameters at $Gr = 2.0$, $Gc = 2.0$, $Pr = 0.7$, $Da = 0.05$, $\chi = 0.5$, $Sc = 0.6$. This has been observed that the effect of M is not much significant, the profile was smooth. This has also been encountered that the velocity is increased as we increased the value of M whereas the temperature and concentration is decreased for the higher values of M. in addition of that this is also found that the effect of M is almost negligible for the characteristic length $\eta \in (4,8)$. This shows that the thermal boundary layer becomes thicker in case of dominating magnetic effect. Whereas the solutal boundary layer is thinner as expected that the following parametric values assumed to be fixed at different level.

Figure 4 shows the effect of schidmt number, the concentration profile is depicted. The Sc is taken 0.2, 0.4, 0.6, 0.8, 1 while fixing the other parameters at $Gr = 2.0$, $Gc = 2.0$, $Pr = 0.7$, $Da = 0.05$, $\chi = 0.5$, $M = 0.5$. Which is conclude that as Sc increased the mass transport decreased. Furthermore, we can say that boundary layer concentration is decreased. This may be the cause of thin vertical moving surface. Numerical results representing the value of skin friction coefficient $U'(0)$ versus Suction parameter (at the vertical surface) for different values of Buoyancy ratio Parameter in the presence and absence of magnetic field is depicted in figure 5.

6. CONCLUSION

In this study the Finite difference study of Hydromagnetic free convective flow over a linearly moving permeable vertical surface in porous medium is undertaken. The model has been transformed into dimensionless form by using the similarity transformation. Our numerical finite difference results indicate that generally:

- Table-I shows the validation of the published results
- Increasing Prandtl number effect significantly the velocity profile whereas its decrease dimensionless heat transfer and increase the mass transfer function values

- Magnetic field can be used to control the flow characteristics along heat and mass transfer.
- Increasing Schmidt number decrease dimensionless mass transfer function values.

LIST OF FIGURES

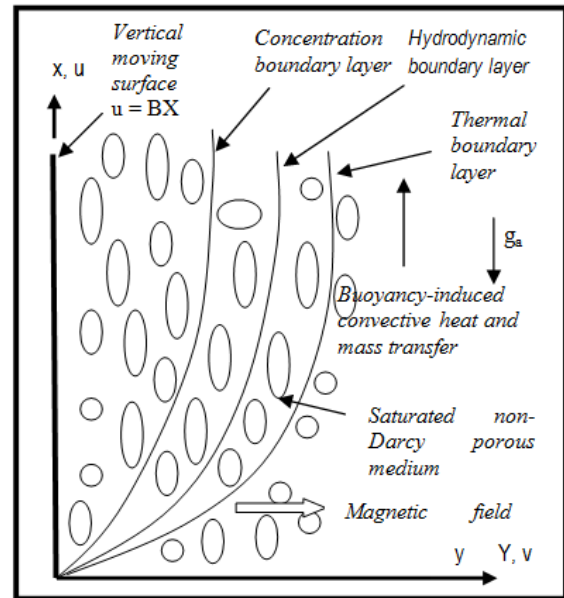


Figure 1: Physical Model of the Problem

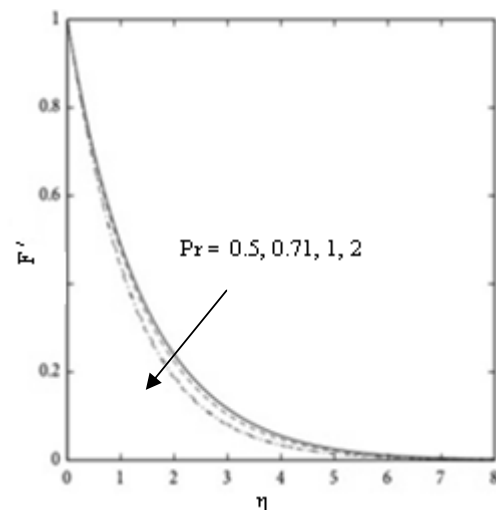


Figure 2 (a) Effect of Pr on Velocity

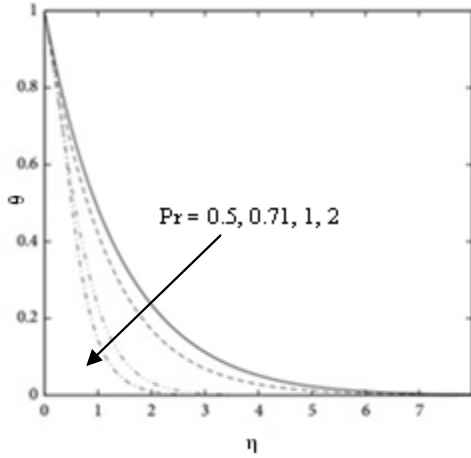


Figure 2(b) Effect of Pr on Temperature

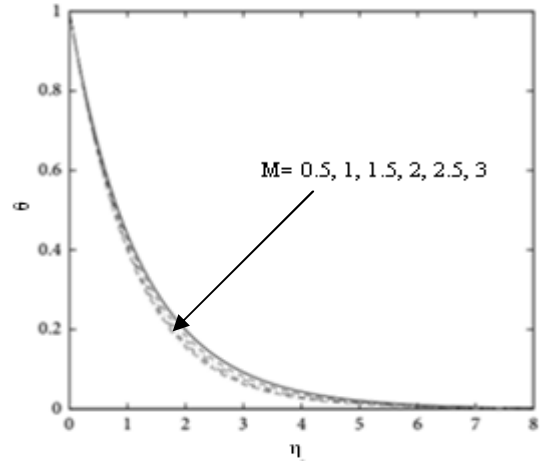


Figure 3(b) Effect of M on Temperature

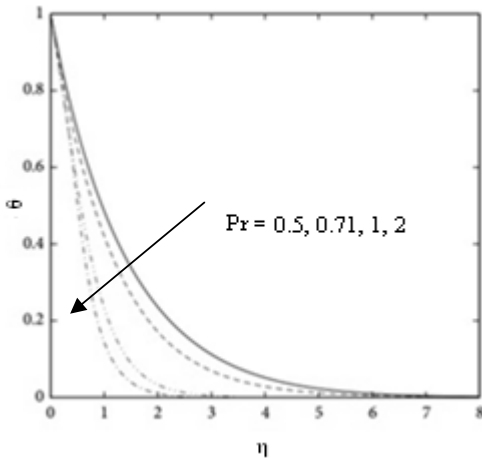


Figure 2(c) Effect of Pr of Concentration

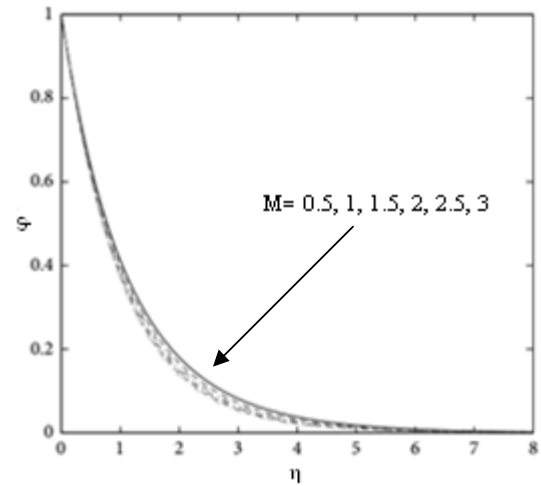


Figure 3(c) Effect of M of Concentration

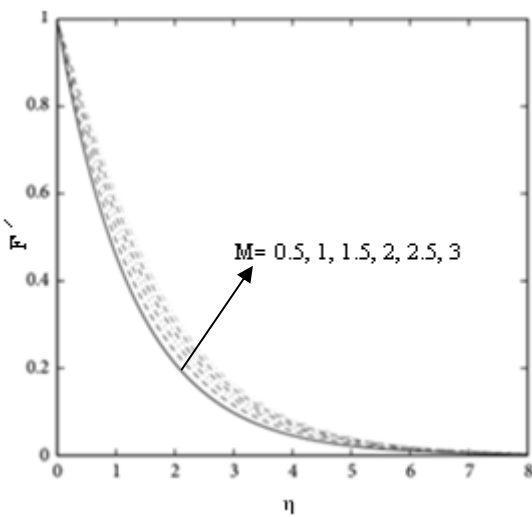


Figure 3 (a) Effect of M on Velocity

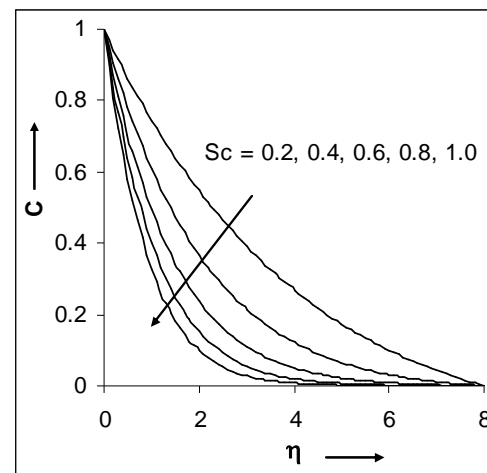


Figure 4: Concentration for different value of Schmidt number

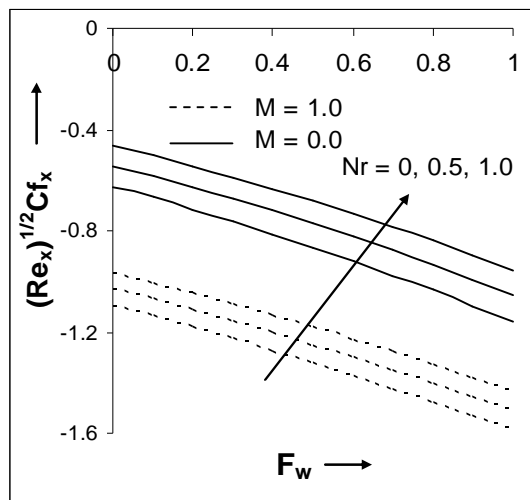


Figure 5: Skin friction for different value of Magnetic number & Buoyancy ratio parameter with the variation in suction parameter

7. ACKNOWLEDGEMENTS

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